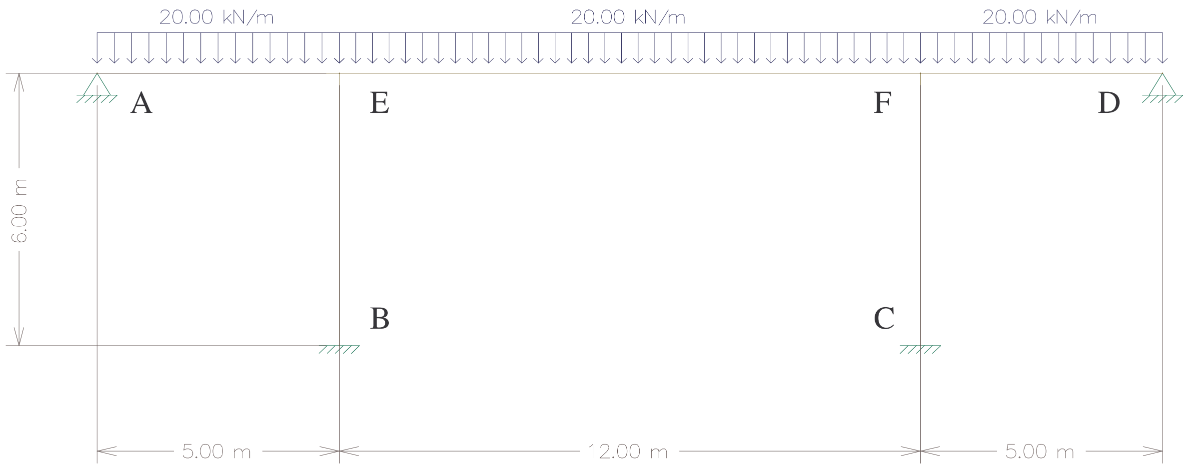
SOLUZIONE DI UN TELAIO IPERSTATICO
- Metodo degli spostamenti -



1. Generalità

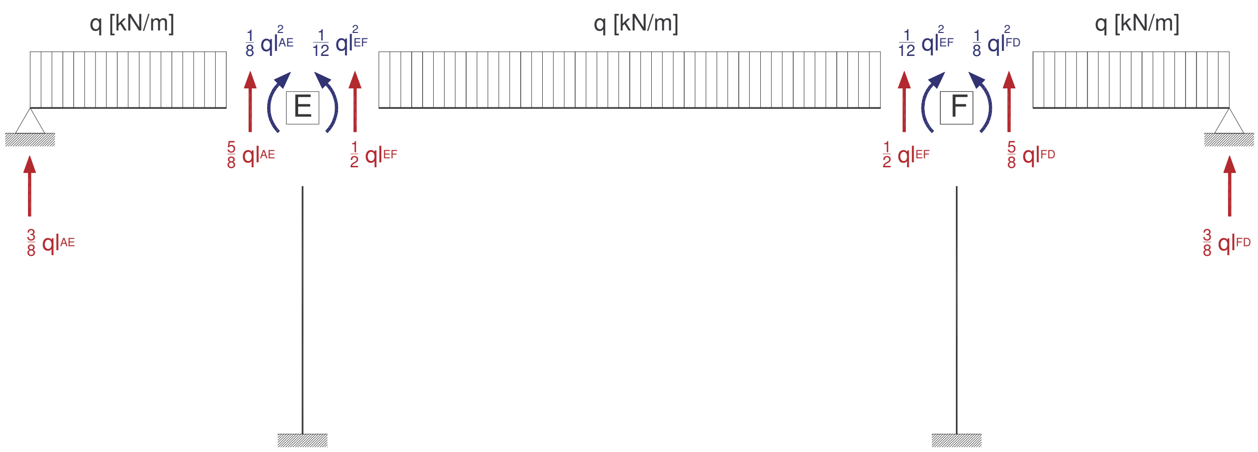
Trave principale: AE – EF – FD
HE400A $J_b = 4.507 \cdot 10^{-4}$ $[m^4]$

Colonne: EB – FC
HE300A $J_c = 1.826 \cdot 10^{-4}$ $[m^4]$

Si considerano gli elementi infinitamente rigidi a forza assiale e tagliante.

2. Sistema “zero” – a nodi bloccati

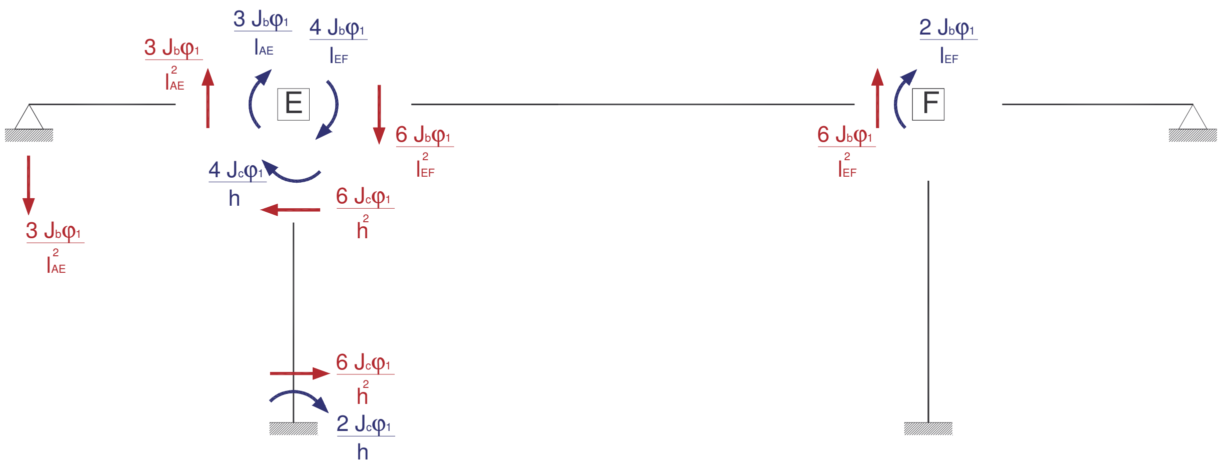
La soluzione del sistema a nodi bloccati con i carichi di progetto avviene per mezzo di abachi di travi semplici di cui siano note le caratteristiche di sollecitazione:



Trave AE e FD = trave incastro cerniera con carico distribuito
Trave EF = trave incastro incastro con carico distribuito

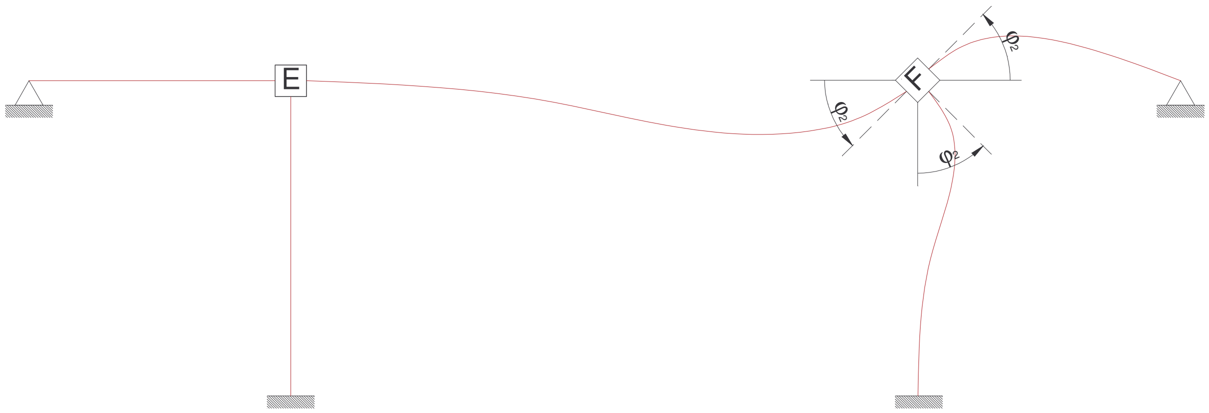
3. Sistema "uno" – a nodi sbloccati (Rotazione del nodo E)

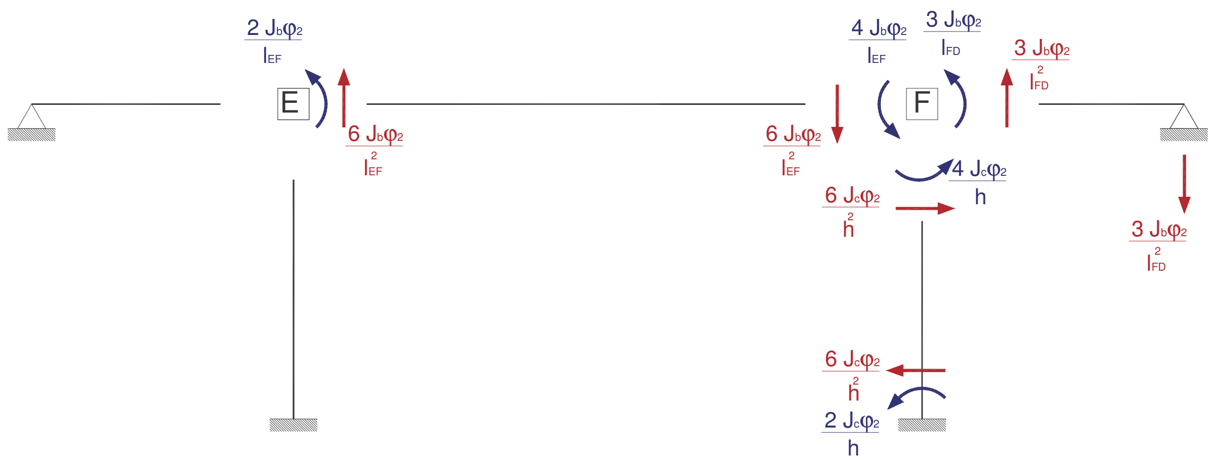
Si impone una rotazione φ_1 al nodo E, e sull'abaco delle "distorsioni nodali" si determinano le forze e i momenti che tale rotazione genera sugli elementi concorrenti nel nodo:



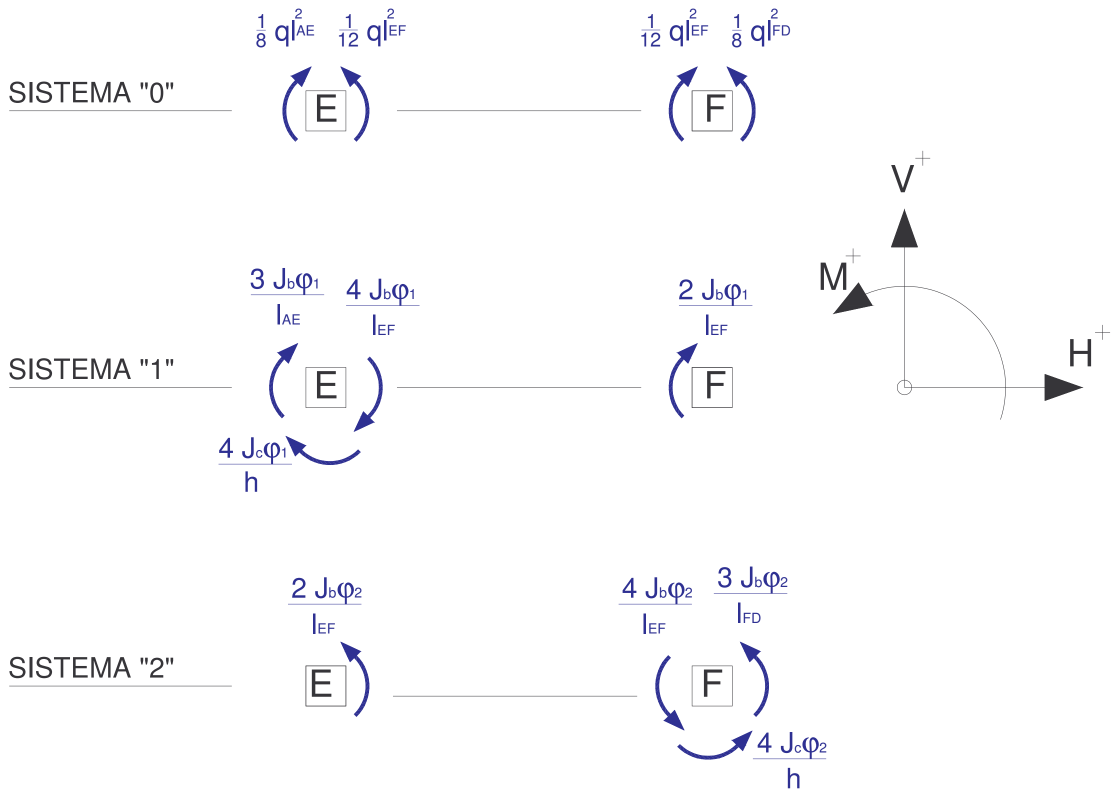
4. Sistema "due" – a nodi sbloccati (Rotazione del nodo F)

Si impone una rotazione φ_2 al nodo F, e sull'abaco delle "distorsioni nodali" si determinano le forze e i momenti che tale rotazione genera sugli elementi concorrenti nel nodo:





5. Determinazione delle rotazioni φ_1 e φ_2



$$\begin{cases} \mu_i = \mu_{i0} + \sum_{k=1}^{n} \mu_{ik} \cdot \varphi_k \\ i = E, F \end{cases}$$

$$\begin{cases} M_E = M_{E0} + M_{E1} \cdot \varphi_1 + M_{E2} \cdot \varphi_2 \\ M_F = M_{F0} + M_{F1} \cdot \varphi_1 + M_{F2} \cdot \varphi_2 \end{cases}$$

<u>Dove:</u>

μ_i — Generica forza esterna applicata direttamente al generico nodo i secondo la direzione dello spostamento ξ_i nel sistema effettivamente assegnato.
(Se $\xi_i = w_i \Rightarrow \mu_i =$forza orizzontale; se $\xi_i = \varphi_i \Rightarrow \mu_i =$momento)

μ_{i0} — Generica sollecitazione indotta nel generico nodo i, nel sistema a nodi bloccati, dai carichi agenti lungo le travi, lungo la direzione dello spostamento ξ_i.

μ_{ik} — Generica sollecitazione indotta nel generico nodo i, nel sistema a nodi sbloccati, nella direzione dello spostamento ξ_i, dovuta allo spostamento generico $\xi_k = 1$ del nodo k.
(Se $\xi_k = v_k = 1 \Rightarrow \xi_i = v_i \Rightarrow \mu_{ik} = T_{ik}$).

n — Numero complessivo delle componenti di spostamento.

I segni dei vari termini del sistema sono determinabili confrontando i momenti sui nodi con la convenzione positiva delle sollecitazioni

$$\begin{cases} 0 = -\frac{1}{8} q l_{AE}^2 + \frac{1}{12} q l_{EF}^2 - \left(\frac{3J_b}{l_{AE}} + \frac{4J_c}{h} + \frac{4J_b}{l_{EF}} \right) \cdot \varphi_1 + \frac{2J_b}{l_{EF}} \cdot \varphi_2 \\ 0 = +\frac{1}{8} q l_{AE}^2 - \frac{1}{12} q l_{EF}^2 - \frac{2J_b}{l_{EF}} \cdot \varphi_1 + \left(\frac{3J_b}{l_{FD}} + \frac{4J_c}{h} + \frac{4J_b}{l_{EF}} \right) \cdot \varphi_2 \end{cases}$$

$l_{AE} = 5.00$ $[m]$
$l_{EF} = 12.00$ $[m]$
$l_{FD} = 5.00$ $[m]$
$h = 6.00$ $[m]$

$J_b = 4.507 \cdot 10^{-4}$ $[m^4]$
$J_c = 1.826 \cdot 10^{-4}$ $[m^4]$

$q = 20$ $[kN/m^2]$

$$\begin{cases} 0 = -62.5 + 240 - (0.00027042 + 0.00012173 + 0.00015023) \cdot \varphi_1 + 0.000075116 \cdot \varphi_2 \\ 0 = +62.5 - 240 - 0.000075116 \cdot \varphi_1 + (0.00027042 + 0.00012173 + 0.00015023) \cdot \varphi_2 \end{cases}$$

$\varphi_1 = \varphi_2 = 379870.908$

6. Determinazione delle caratteristiche di sollecitazione M e V

I momenti sono espressi in $[kN \cdot m]$

NODO E

$M_{EA} = -\frac{q l_{EA}^2}{8} - \frac{3J_b}{l_{EA}} \cdot \varphi_1 = -62.5 - 102.72 = -165.22$ rotazione negativa

$M_{EF} = \frac{q l_{EF}^2}{12} - \frac{4J_b}{l_{EF}} \cdot \varphi_1 + \frac{2J_b}{l_{EF}} \cdot \varphi_2 = 211.47$ rotazione positiva

$M_{EB} = -\frac{4J_c}{h} \cdot \varphi_1 = -46.24$ rotazione negativa

NODO F

$M_{FD} = \frac{q l_{FD}^2}{8} + \frac{3J_b}{l_{FD}} \cdot \varphi_2 = 62.5 + 102.72 = 165.22$ rotazione positiva

$M_{FE} = -\frac{q l_{FE}^2}{12} - \frac{2J_b}{l_{FE}} \cdot \varphi_1 + \frac{4J_b}{l_{FE}} \cdot \varphi_2 = -211.47$ rotazione negativa

$M_{FB} = \frac{4J_c}{h} \cdot \varphi_2 = 46.24$ rotazione positiva

NODO B

$$M_{BE} = -\frac{2J_c}{h} \cdot \varphi_1 = -23.12$$ rotazione negativa

NODO C

$$M_{CF} = +\frac{2J_c}{h} \cdot \varphi_2 = 23.12$$ rotazione positiva

I tagli sono espressi in $[kN]$

NODO E

$$V_{EA} = \frac{5}{8} q l_{EA} + \frac{3J_b}{l_{EA}^2} \cdot \varphi_1 = 83.04$$ verso positivo

$$V_{EF} = \frac{1}{2} q l_{EF} - \frac{6J_b}{l_{EF}^2} \cdot \varphi_1 + \frac{6J_b}{l_{EF}^2} \cdot \varphi_2 = 120$$ verso positivo

$$V_{EB} = -\frac{6J_c}{h^2} \cdot \varphi_1 = -11.56$$ verso negativo

NODO F

$$V_{FD} = \frac{5}{8} q l_{FD} + \frac{3J_b}{l_{FD}^2} \cdot \varphi_2 = 83.04$$ verso positivo

$$V_{FE} = \frac{1}{2} q l_{FE} + \frac{6J_b}{l_{FE}^2} \cdot \varphi_1 - \frac{6J_b}{l_{FE}^2} \cdot \varphi_2 = 120$$ verso positivo

$$V_{FC} = \frac{6J_c}{h^2} \cdot \varphi_2 = 11.56$$ verso positivo

NODO B

$$V_{BE} = \frac{6J_c}{h^2} \cdot \varphi_1 = 11.56$$ verso positivo

NODO C

$$V_{CF} = -\frac{6J_c}{h^2} \cdot \varphi_2 = -11.56$$ verso negativo

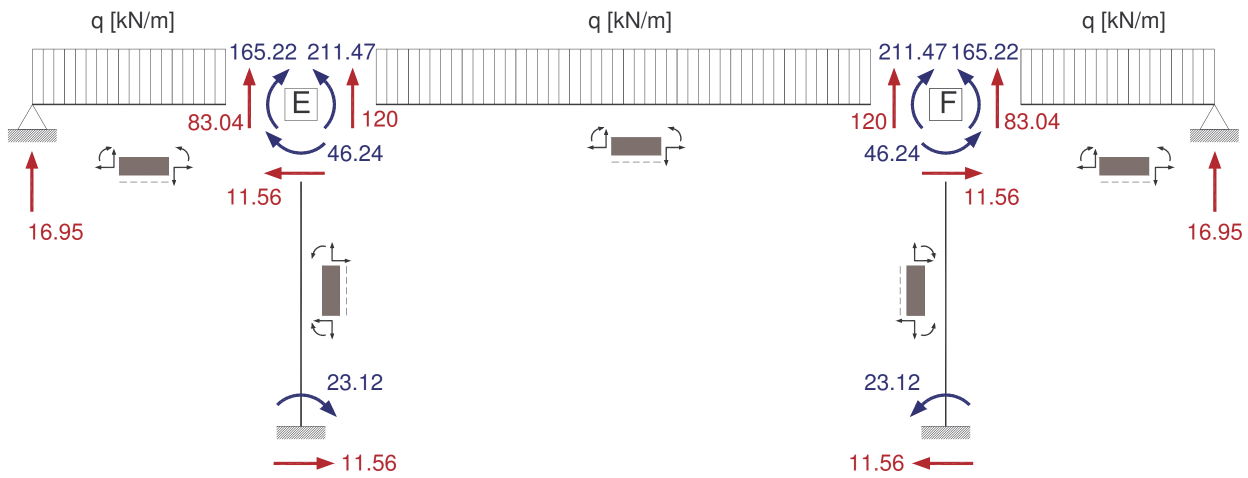
NODO A

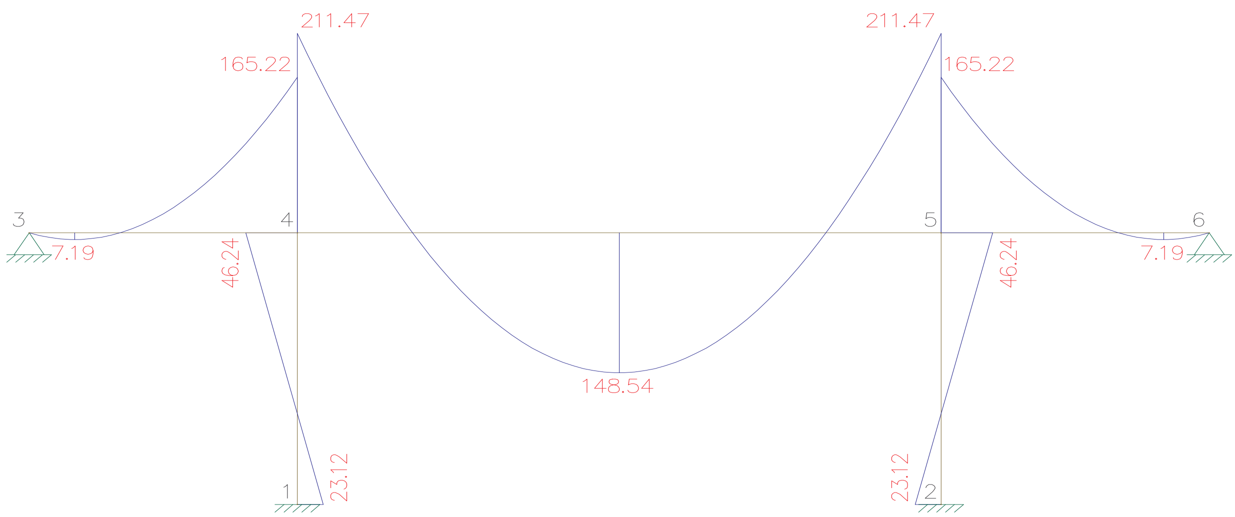
$$V_{AE} = \frac{3}{8} q l_{AE} - \frac{3J_b}{l_{AE}^2} \cdot \varphi_1 = 16.95$$ verso positivo

NODO D

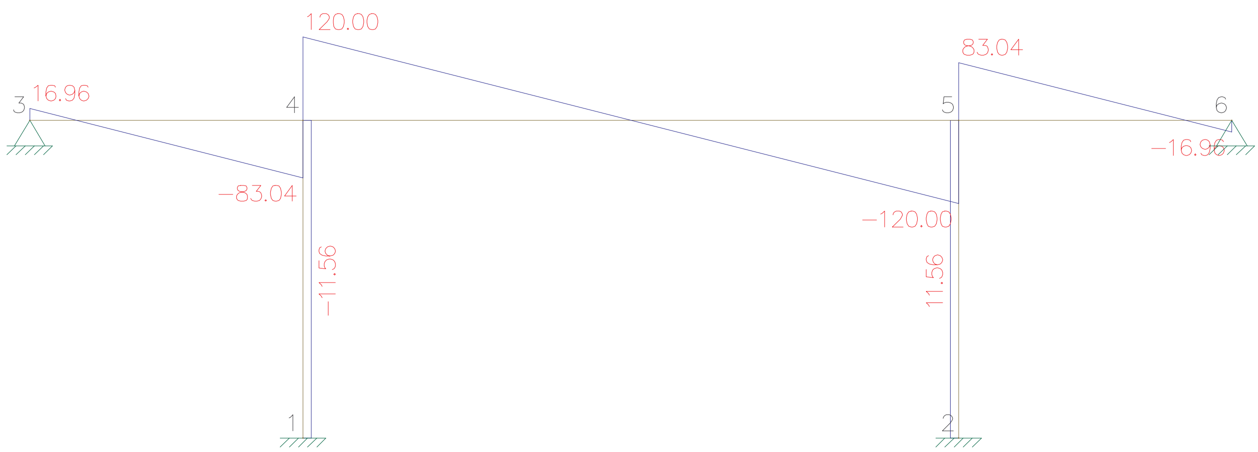
$$V_{DF} = \frac{3}{8} q l_{DF} - \frac{3J_b}{l_{DF}^2} \cdot \varphi_2 = 16.95$$ verso positivo

Da ciò risulta in accordo con la convenzione positiva:

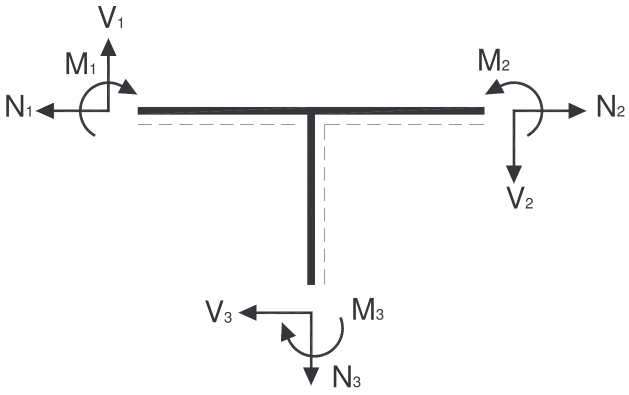


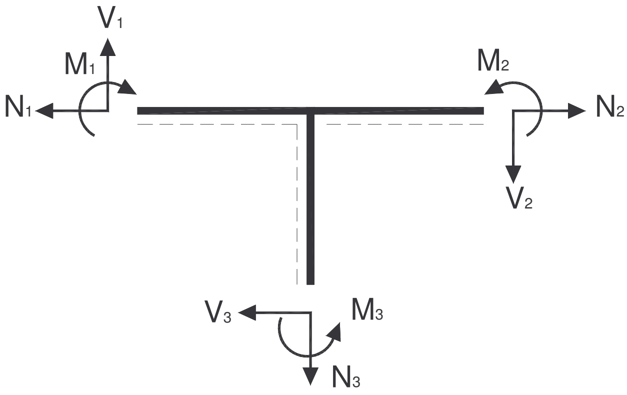


$$M^{+}_{\max,EF} = -M_{EF} + V_{EF} \cdot z - q \cdot \frac{z^2}{2} = -211.47 + 120 \cdot 6 - \frac{20 \cdot 6^2}{2} = 148.54$$



Verifica delle sollecitazioni nei nodi





$$\begin{cases} N_1 - N_2 + V_3 = 0 \\ V_1 - V_2 - N_3 = 0 \\ M_1 - M_2 + M_3 = 0 \end{cases}$$

$$\begin{cases} N_1 - N_2 + V_3 = 0 \\ V_1 - V_2 - N_3 = 0 \\ M_1 - M_2 - M_3 = 0 \end{cases}$$

NODO E

La prima equazione non può essere ancora risolta perché possiede due incognite:
Dalla seconda si ottengono le forze normali sui pilastri:

$$\left.\begin{matrix} V_{EA} - V_{EF} - N_{EB} = 0 \\ -83.04 - 120 - N_{EB} = 0 \end{matrix}\right\} \qquad N_{EB} = -203.04 \qquad [kN]$$

$$M_{EA} - M_{EF} + M_{EB} = 0$$
$$-165.22 - (-211.47) + (-46.24) = 0$$

NODO F

La prima equazione non può essere ancora risolta perché possiede due incognite:
Dalla seconda si ottengono le forze normali sui pilastri:

$$\left.\begin{matrix} V_{FE} - V_{FD} - N_{FC} = 0 \\ -120 - 83.04 - N_{FC} = 0 \end{matrix}\right\} \qquad N_{FC} = -203.04 \qquad [kN]$$

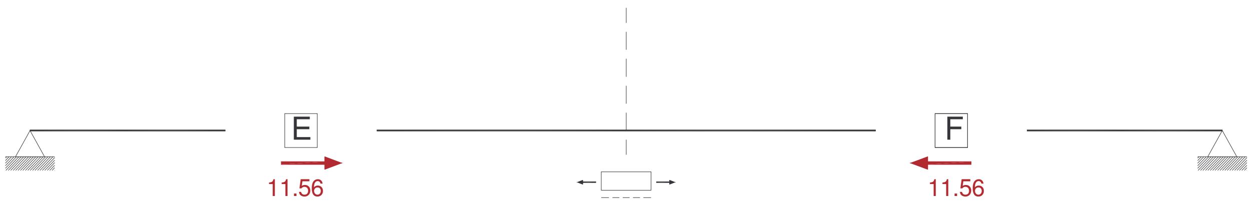
$$M_{FE} - M_{FD} - M_{FC} = 0$$
$$-211.47 - (-165.22) - (-46.24) = 0$$

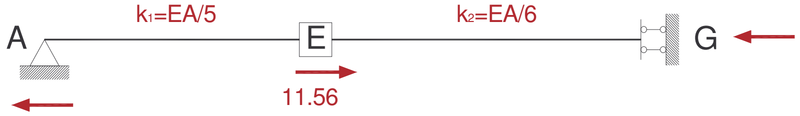
7. Determinazione delle caratteristiche di sollecitazione N

Il precedente metodo porta alla determinazione delle sollecitazioni sui pilastri:

$$N_{EB} = -203.04 \qquad [kN]$$
$$N_{FC} = -203.04 \qquad [kN]$$

La forza di taglio sui pilastri è uguale e contraria alla forza normale sul traverso:





Si ha, in termini di rigidezza assiale:

$$k_{tot} = k_{AE} + k_{EFG} = \frac{E \cdot A}{l_{AE}} + \frac{E \cdot A}{l_{EG}} = \frac{E \cdot A}{5} + \frac{E \cdot A}{6} = \frac{11}{30} \cdot E \cdot A$$

$$N_{AE} : k_{AE} = P_E : k_{tot} \qquad |N_{AE}| = \frac{P_E \cdot k_{AE}}{k_{tot}} = \frac{11.56 \cdot \frac{E \cdot A}{5}}{\frac{11}{30} \cdot E \cdot A} = 6.31 \qquad [kN]$$

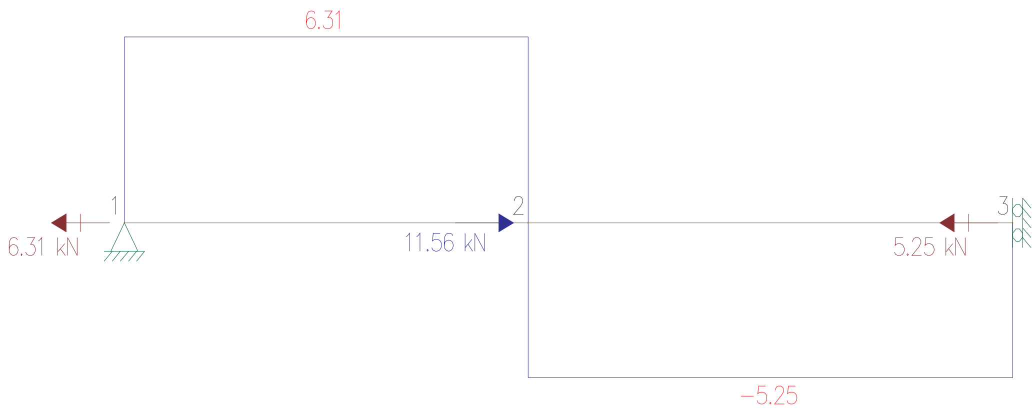
$$N_{EG} : k_{EG} = P_E : k_{tot} \qquad |N_{EG}| = \frac{P_E \cdot k_{EG}}{k_{tot}} = \frac{11.56 \cdot \frac{E \cdot A}{6}}{\frac{11}{30} \cdot E \cdot A} = 5.25 \qquad [kN]$$

Da cui:

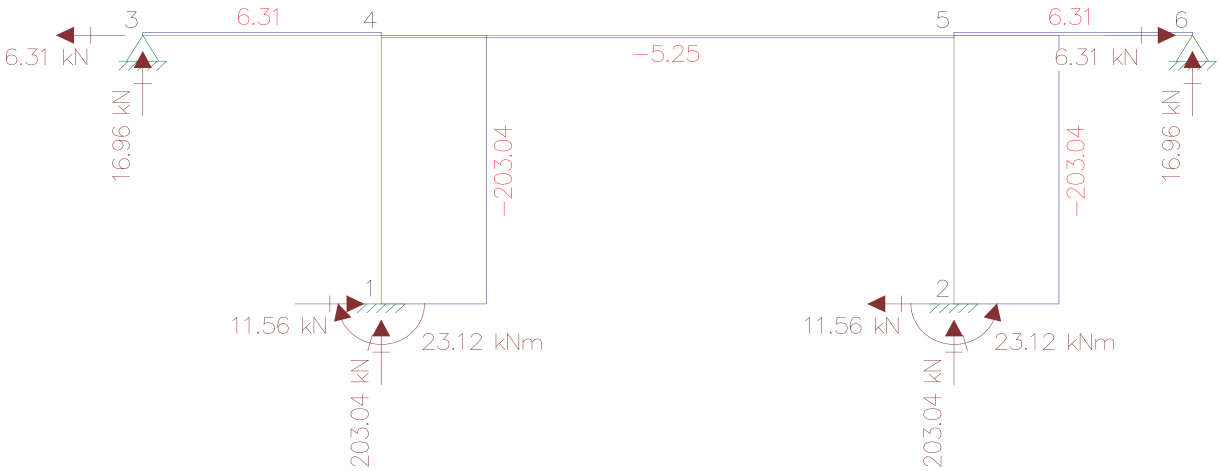
$$N_{AE} = +6.31 \qquad [kN]$$

$$N_{EG} = -5.25 \qquad [kN]$$

Porzione di traverso



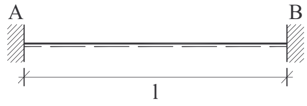
Telaio integrale



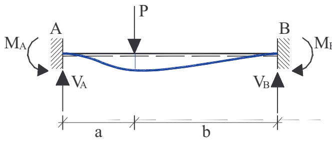
Fine Esempio

SOLUZIONI DI TRAVI ELEMENTARI VARIAMENTE CARICATE

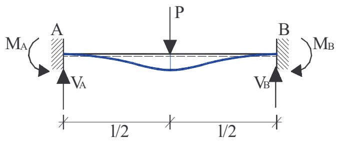
Travi con doppio incastro



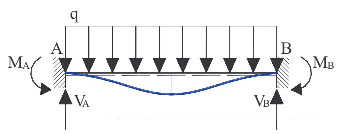
CARICHI ESTERNI



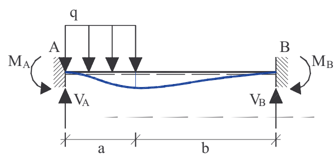
$$V_A = \frac{Pb^2}{l^3}(l+2a); \quad V_B = \frac{Pa^2}{l^3}(l+2b)$$
$$M_A = -\frac{Pab^2}{l^2}; \quad M_B = \frac{Pa^2b}{l^2}$$



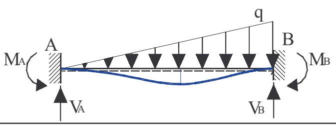
$$V_A = V_B = \frac{P}{2}$$
$$M_A = M_B = \frac{Pl}{8}$$



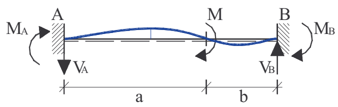
$$V_A = V_B = \frac{ql}{2}$$
$$M_A = M_B = \frac{ql^2}{12}$$



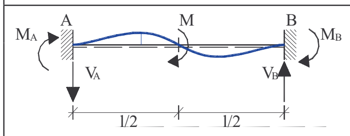
$$V_A = \frac{qa}{2}\left(2 - \frac{2a^2}{l^2} + \frac{a^3}{l^3}\right);$$
$$V_B = \frac{qa^3}{2l^2}\left(2 - \frac{a}{l}\right)$$
$$M_A = -qa^2\left(\frac{1}{2} - \frac{2a}{3l} + \frac{a^2}{4l^2}\right);$$
$$M_B = -qa^2\left(\frac{a}{3l} - \frac{a^2}{4l^2}\right)$$



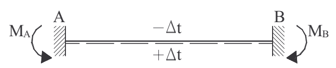
$$V_A = \frac{3}{20}ql; \quad V_B = \frac{7}{20}ql$$
$$M_A = \frac{1}{30}ql^2; \quad M_B = \frac{1}{20}ql^2$$



$$V_A = -V_B = -\frac{6Mab}{l^3}$$
$$M_A = M\frac{b}{l}\left(2 - 3\frac{b}{l}\right);$$
$$M_B = -M\frac{a}{l}\left(2 - 3\frac{a}{l}\right)$$

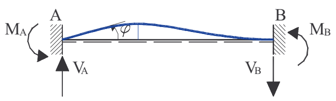


$$V_A = -V_B = \frac{3}{2}\frac{M}{l}$$
$$M_A = M_B = \frac{M}{4}$$

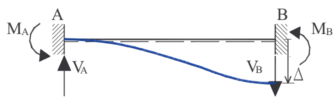


$$M_A = -M_B = \frac{\alpha \Delta t EJ}{h}$$

DISTORSIONI VINCOLARI

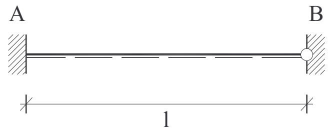


$$V_A = -V_B = \frac{6\varphi EJ}{l^2}$$
$$M_A = \frac{4\varphi EJ}{l}; \quad M_B = \frac{2\varphi EJ}{l}$$

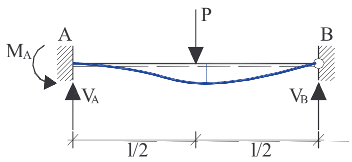


$$V_A = -V_B = \frac{12\Delta EJ}{l^3}$$
$$M_A = M_B = \frac{6\Delta EJ}{l^2}$$

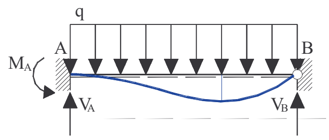
Travi con incastro e cerniera



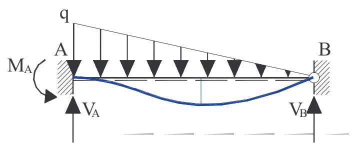
CARICHI ESTERNI



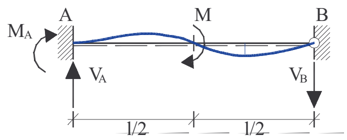
$$V_A = \frac{11}{16}P; \quad V_B = \frac{5}{16}P$$
$$M_A = \frac{3}{16}Pl$$



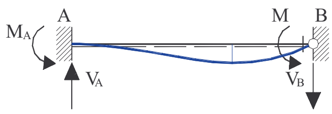
$$V_A = \frac{5}{8}ql; \quad V_B = \frac{3}{8}ql$$
$$M_A = \frac{ql^2}{8}$$



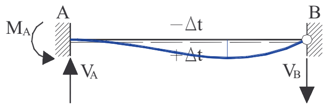
$$V_A = \frac{2}{5}ql; \quad V_B = \frac{ql}{10}$$
$$M_A = \frac{ql^2}{15}$$



$$V_A = -V_B = \frac{9}{8}\frac{M}{l}$$
$$M_A = \frac{M}{8}$$

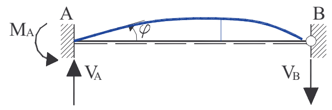


$$V_A = -V_B = \frac{3}{2}\frac{M}{l}$$
$$M_A = \frac{M}{2}$$

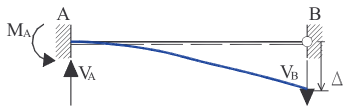


$$V_A = -V_B = \frac{3}{2}\frac{\alpha \Delta t EJ}{lh}$$
$$M_A = \frac{3}{2}\frac{\alpha \Delta t EJ}{h}$$

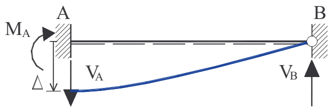
DISTORSIONI VINCOLARI



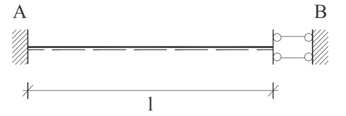
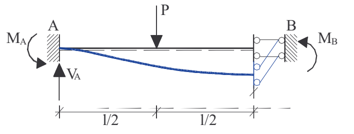
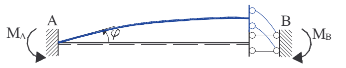
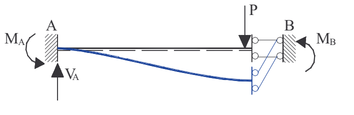
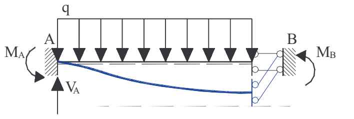
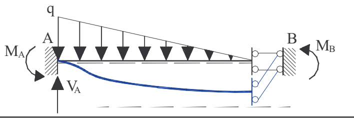
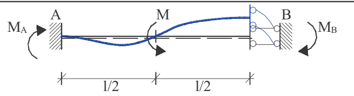
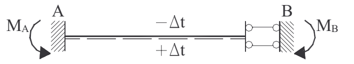
$$V_A = -V_B = \frac{3\varphi EJ}{l^2}$$
$$M_A = \frac{3\varphi EJ}{l}$$



$$V_A = -V_B = \frac{3\Delta EJ}{l^3}$$
$$M_A = \frac{3\Delta EJ}{l^2}$$



$$V_A = -V_B = \frac{3\Delta EJ}{l^3}$$
$$M_A = \frac{3\Delta EJ}{l^2}$$

Travi con incastro e doppio pendolo			
CARICHI ESTERNI		**DISTORSIONI VINCOLARI**	
	$V_A = P$ $M_A = \frac{3}{8}Pl$; $M_B = \frac{Pl}{8}$		$M_A = -M_B = \frac{\varphi EJ}{l}$
	$V_A = P$ $M_A = M_B = \frac{Pl}{2}$		
	$V_A = ql$ $M_A = \frac{ql^2}{3}$; $M_B = \frac{ql^2}{6}$		
	$V_A = \frac{ql}{2}$ $M_A = \frac{ql^2}{8}$; $M_B = \frac{ql^2}{24}$		
	$M_A = M_B = \frac{M}{2}$		
	$M_A = -M_B = \frac{\alpha \Delta t EJ}{h}$		