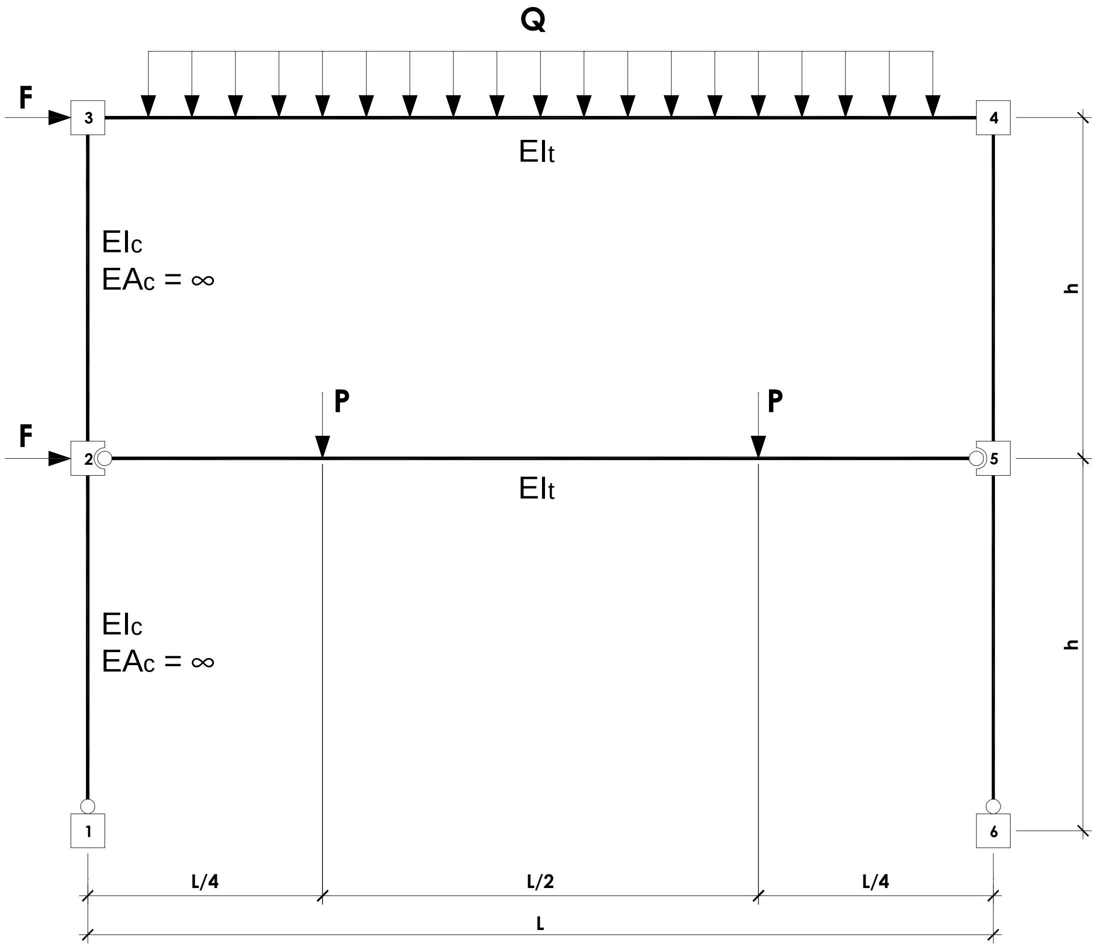
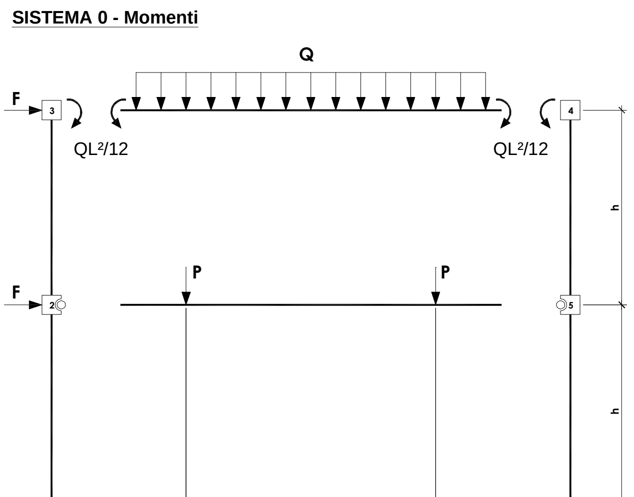
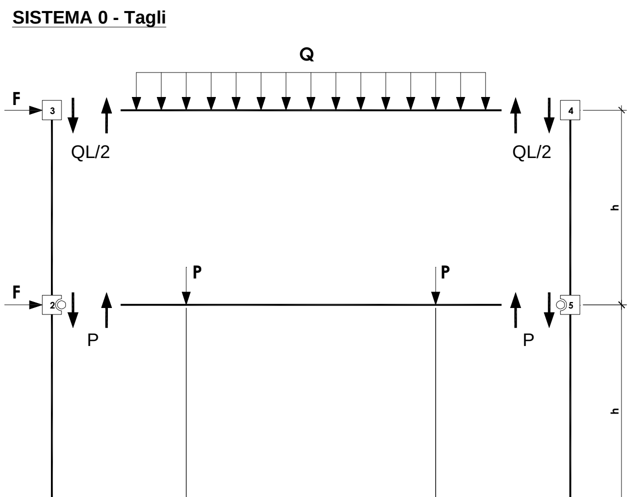


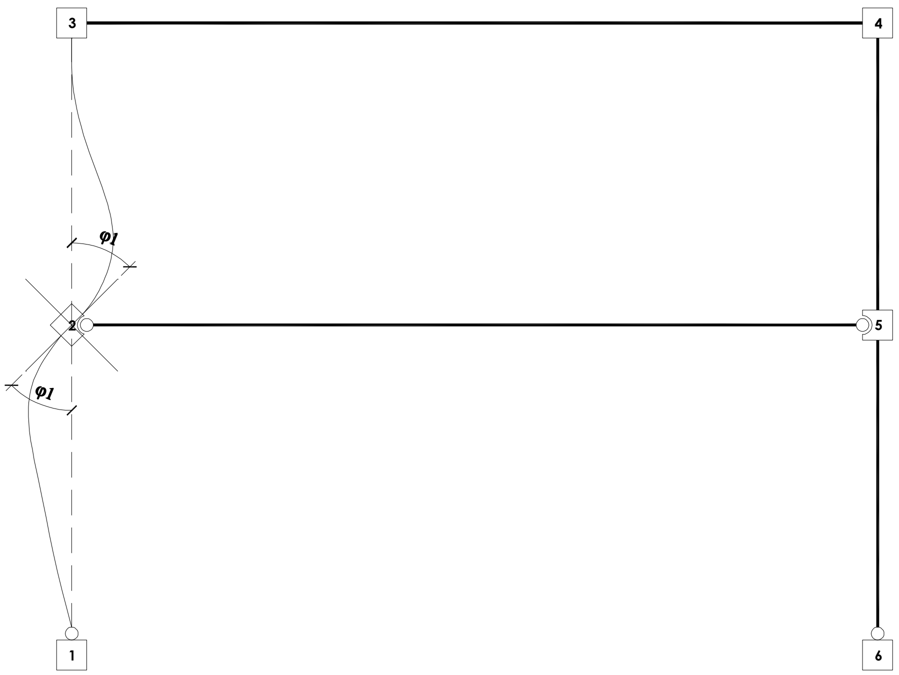
Risoluzione del "sistema a nodi fissi":
La soluzione di tale sistema si basa sull'individuazione delle azioni che le travi caricare dalle azioni esterne "Q" e "P" trasferiscono ai nodi di estremità. Si noti che i carichi "F" essendo direttamente agenti sui nodi non rientrano nelle soluzioni del sistema zero.



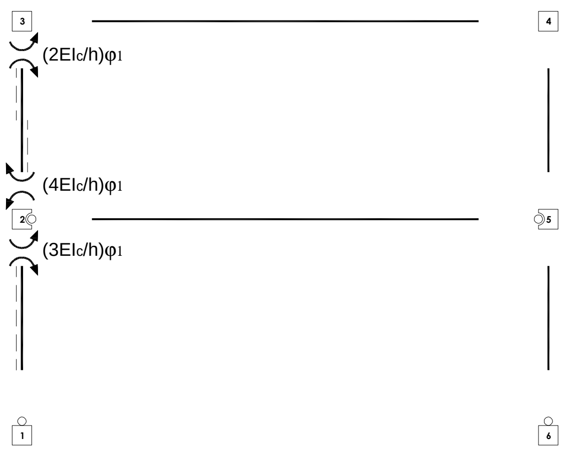


Risoluzione dei "sistemi a nodi spostabili":

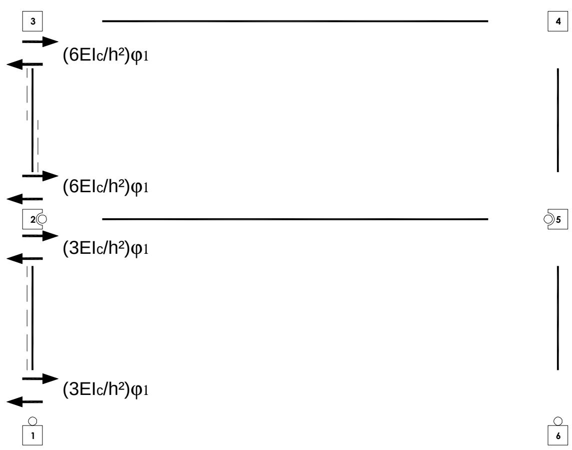
SISTEMA 1 - Rotazione del nodo 2



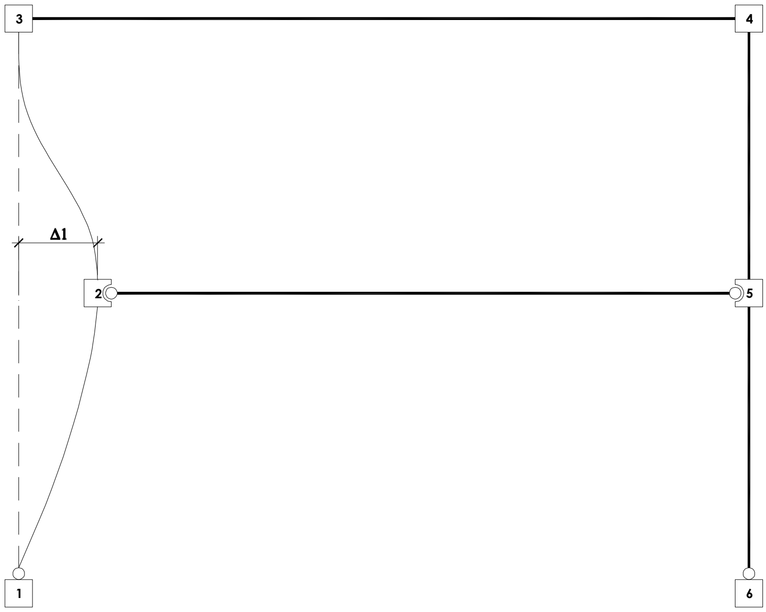
SISTEMA 1 - Momenti



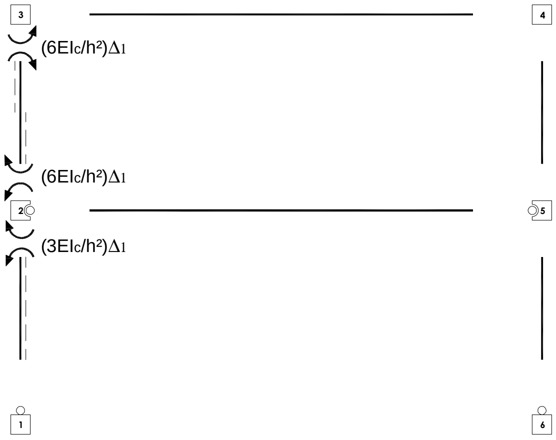
SISTEMA 1 - Tagli



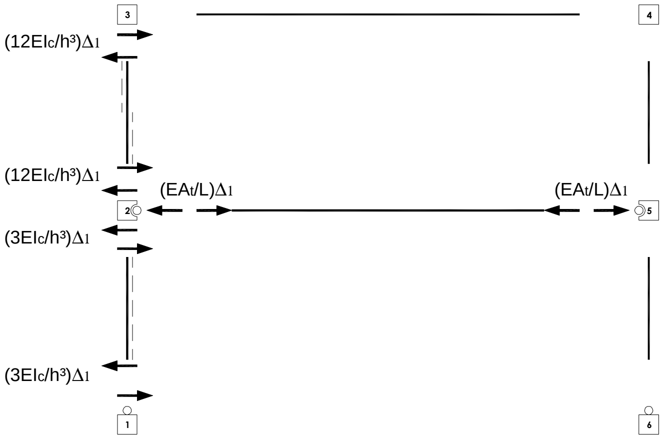
SISTEMA 2 - Traslazione del nodo 2



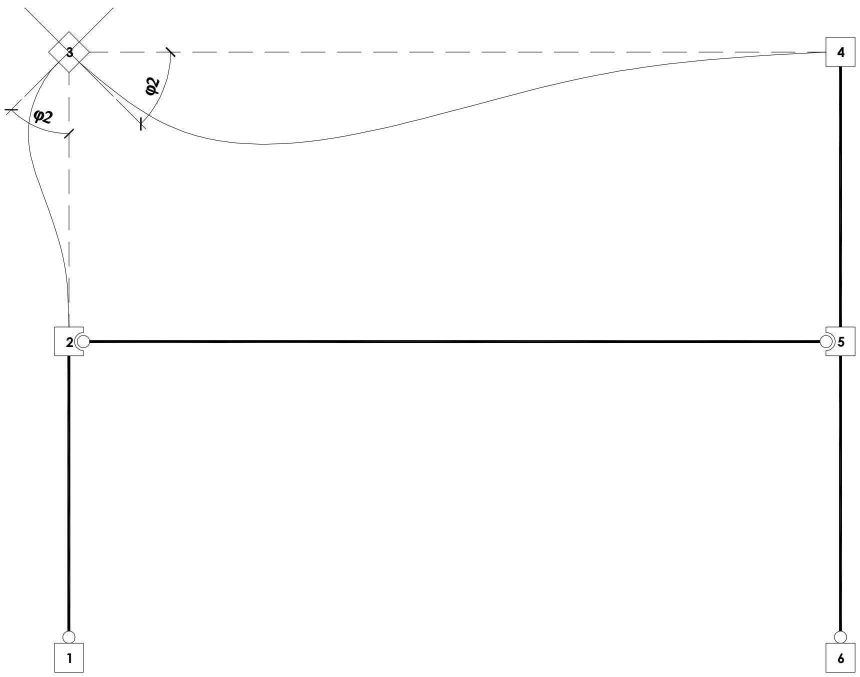
SISTEMA 2 - Momenti



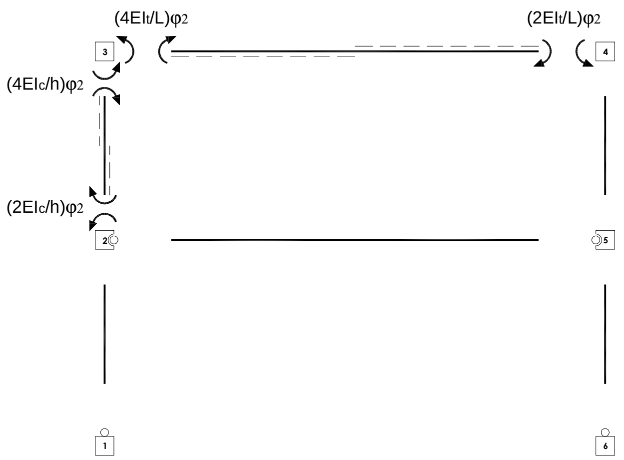
SISTEMA 2 - Tagli



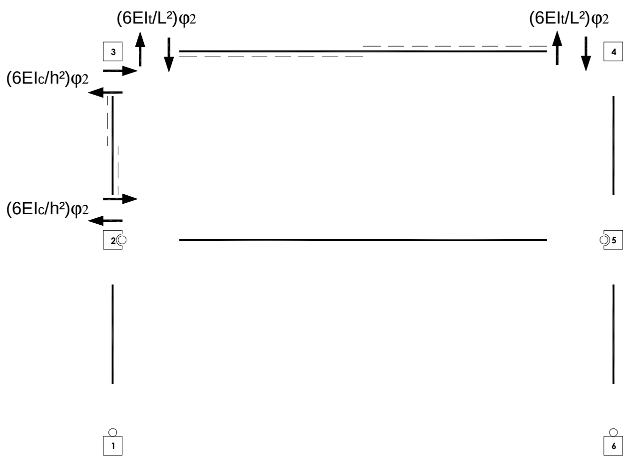
SISTEMA 3 - Rotazione del nodo 3



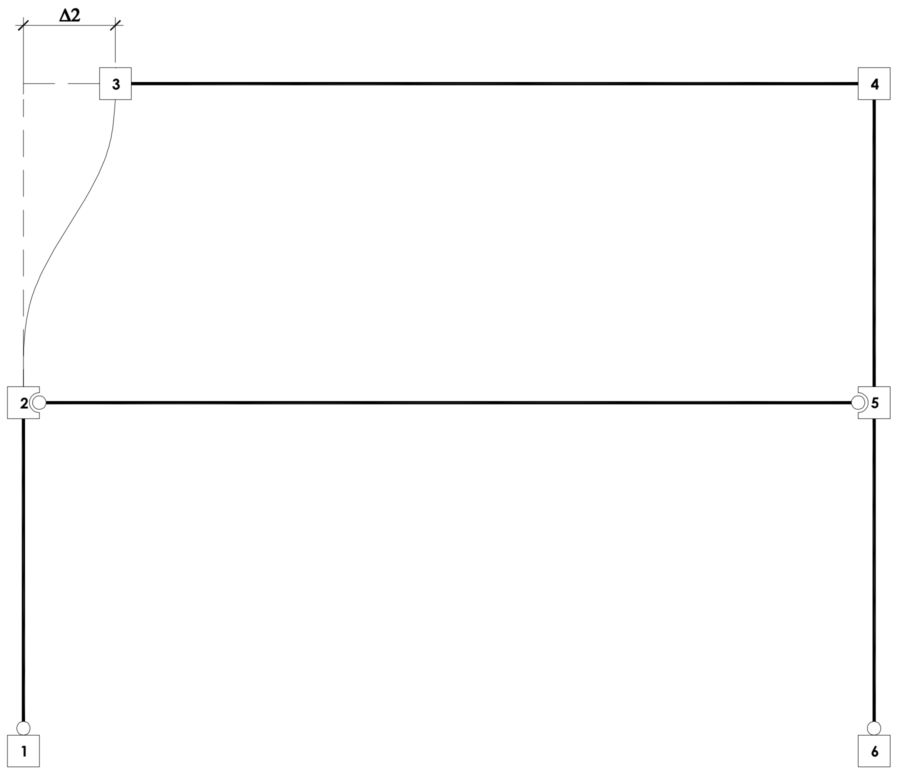
SISTEMA 3 - Momenti



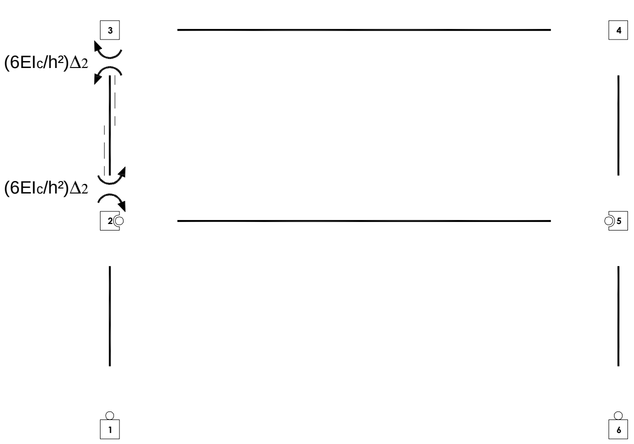
SISTEMA 3 - Tagli



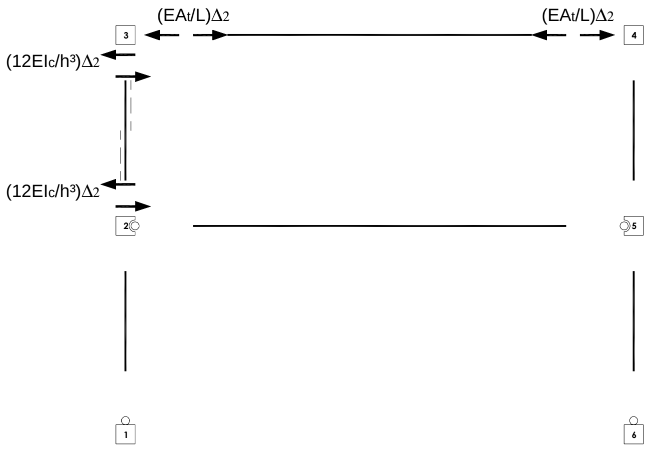
SISTEMA4 - Traslazione del nodo 3



SISTEMA 4 - Momenti

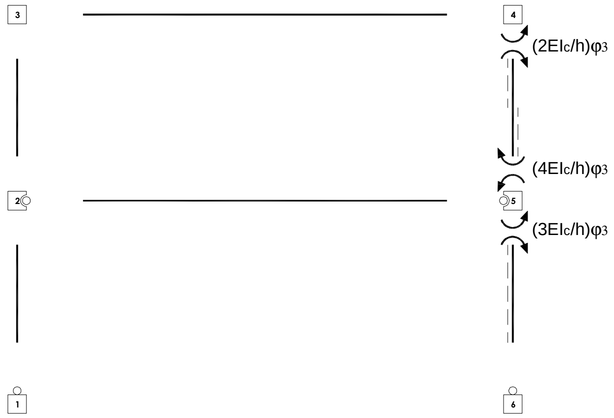


SISTEMA 4 - Tagli

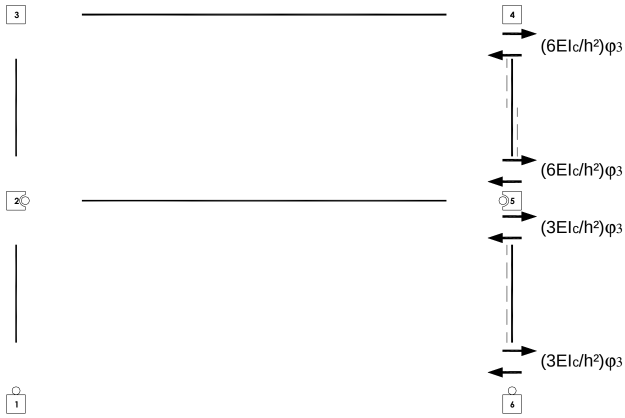


SISTEMA 5 - Rotazione del nodo 5

SISTEMA 5 - Momenti

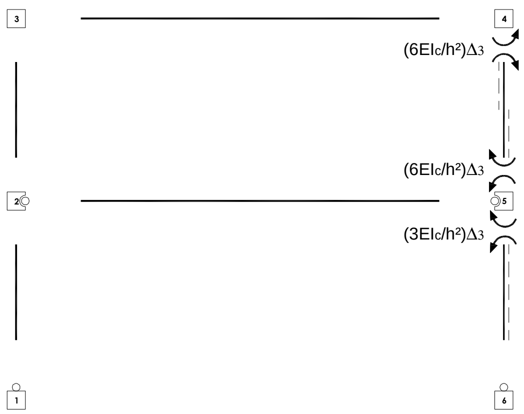


SISTEMA 5 - Tagli

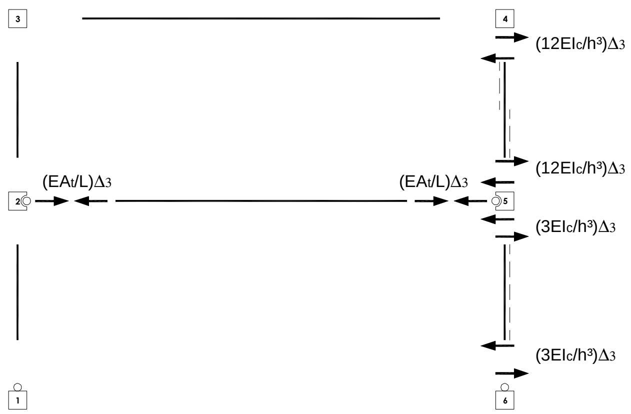


SISTEMA 6 - Traslazione del nodo 5

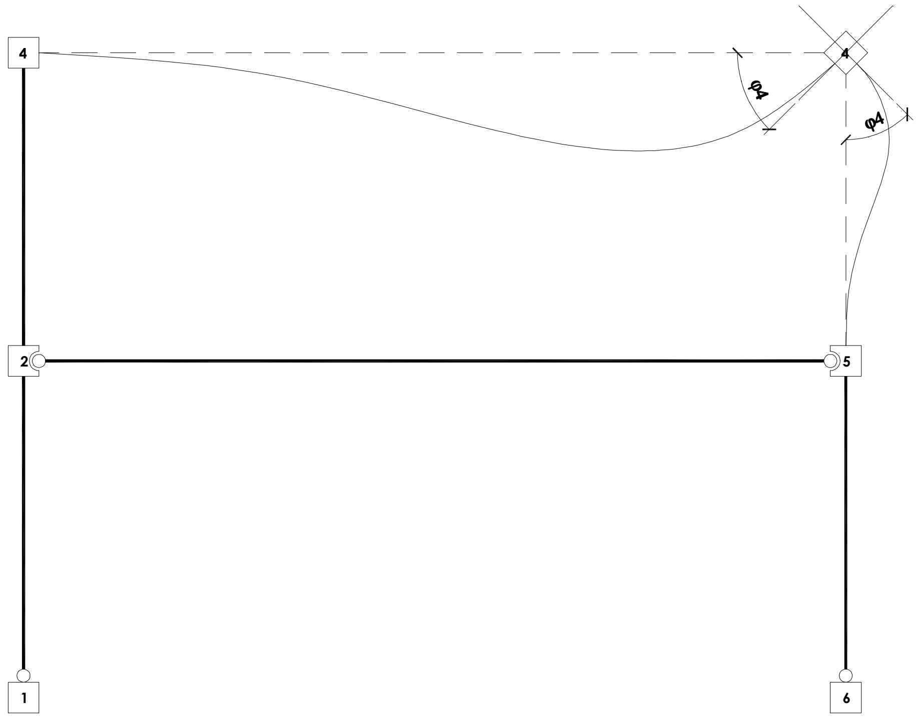
SISTEMA 6 - Momenti



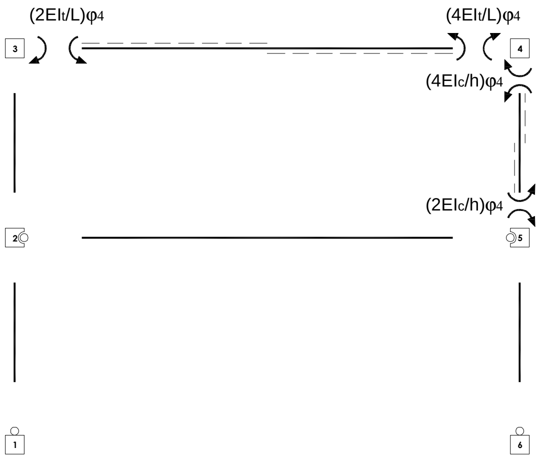
SISTEMA 6 - Tagli



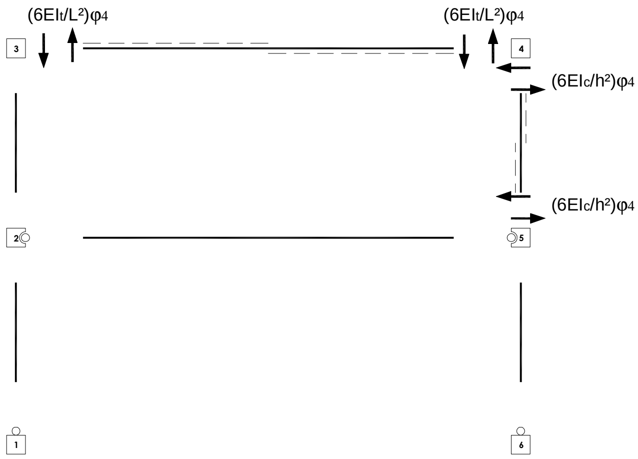
SISTEMA 7 - Rotazione del nodo 4



SISTEMA 7 - Momenti

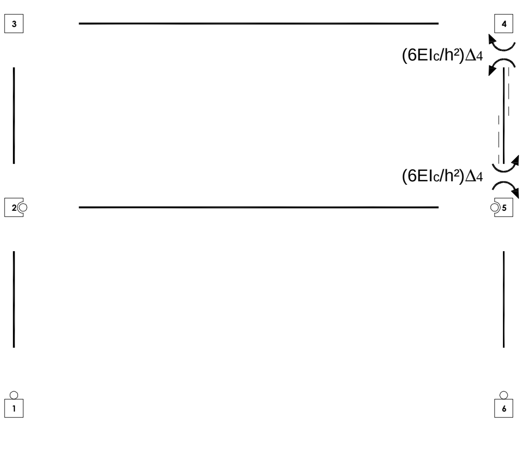


SISTEMA 7 - Tagli

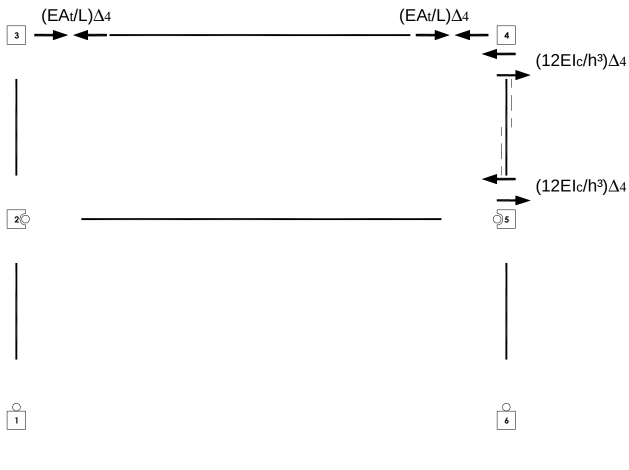


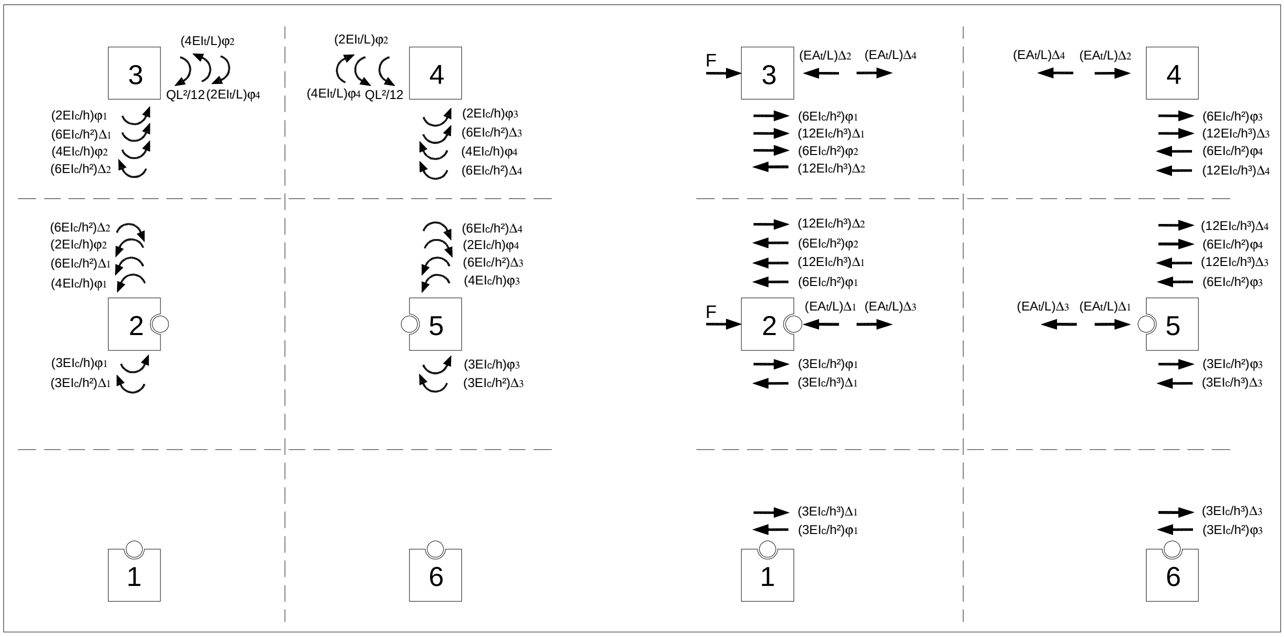
SISTEMA8 - Traslazione del nodo 4

SISTEMA 8 - Momenti



SISTEMA 8 - Tagli





Si definiscono le seguenti relazioni:

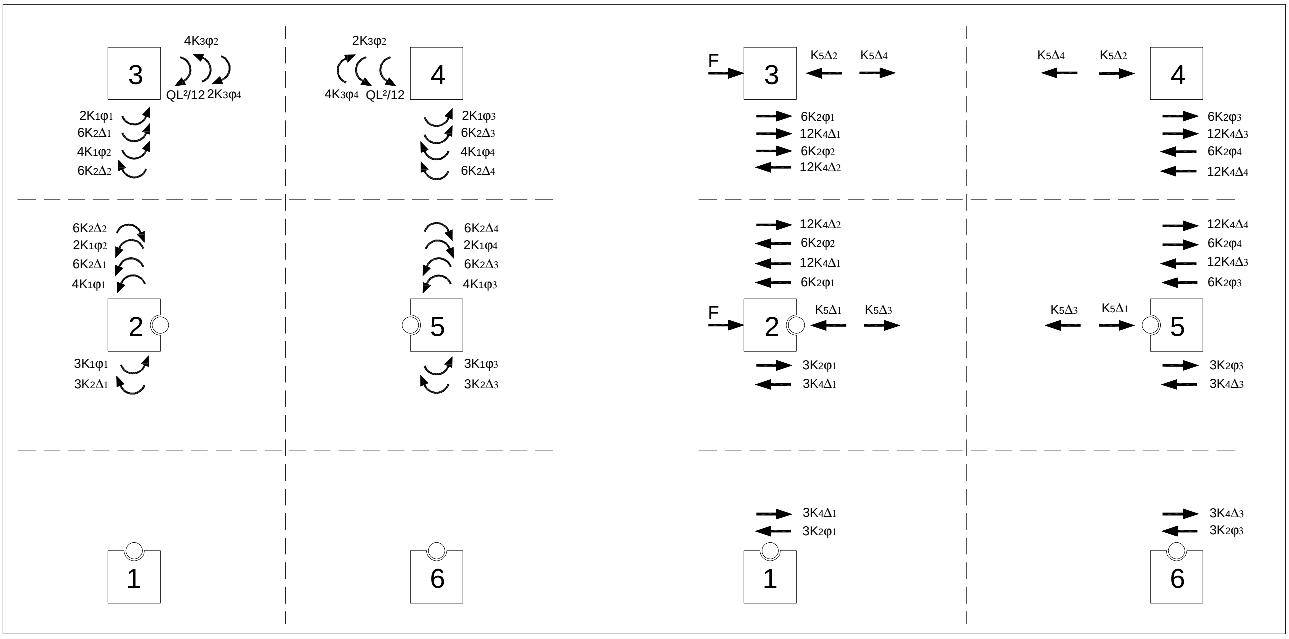
$$K_1 = \frac{EI_c}{h}$$

$$K_2 = \frac{EI_c}{h^2}$$

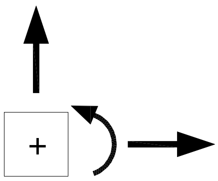
$$K_3 = \frac{EI_t}{L}$$

$$K_4 = \frac{EI_c}{h^3}$$

$$K_5 = \frac{EA_t}{L}$$



La soluzione del sistema è legata alla determinazione degli otto spostamenti incogniti dei nodi 2 – 3 – 4 – 5. La convenzione positiva delle forze e dei momenti agenti sul nodo è indicata nella figura seguente:



$$
\begin{cases}
0=[3K_1+4K_1]\cdot\varphi_1+[6K_2-3K_2]\cdot\Delta_1+2K_1\cdot\varphi_2-6K_2\cdot\Delta_2 & Nodo\_2\\
0=F+[-6K_2+3K_2]\cdot\varphi_1-[12K_4+3K_4+K_5]\cdot\Delta_1-6K_2\cdot\varphi_2+12K_4\cdot\Delta_2+K_5\cdot\Delta_3 & Nodo\_2\\
0=-\dfrac{Q\cdot L^2}{12}+2K_1\cdot\varphi_1+6K_2\cdot\Delta_1+4[K_1+K_3]\cdot\varphi_2-6K_2\cdot\Delta_2-2K_3\cdot\varphi_4 & Nodo\_3\\
0=F+6K_2\cdot\varphi_1+12K_4\cdot\Delta_1+6K_2\cdot\varphi_2-[12K_4+K_5]\cdot\Delta_2+K_5\cdot\Delta_4 & Nodo\_3\\
0=[3K_1+4K_1]\cdot\varphi_3+[6K_2-3K_2]\cdot\Delta_3-2K_1\cdot\varphi_4-6K_2\cdot\Delta_4 & Nodo\_5\\
0=K_5\cdot\Delta_1+[-6K_2+3K_2]\cdot\varphi_3-[12K_4+3K_4+K_5]\cdot\Delta_3+6K_2\cdot\varphi_4+12K_4\cdot\Delta_4 & Nodo\_5\\
0=\dfrac{Q\cdot L^2}{12}+2K_3\cdot\varphi_2+2K_1\cdot\varphi_3+6K_2\cdot\Delta_3-4[K_1+K_3]\cdot\varphi_4-6K_2\cdot\Delta_4 & Nodo\_4\\
0=K_5\cdot\Delta_2+6K_2\cdot\varphi_3+12K_4\cdot\Delta_3-6K_2\cdot\varphi_4-[12K_4+K_5]\cdot\Delta_4 & Nodo\_4
\end{cases}
$$

$$[\mathbf{F}_{ij}]=[\mathbf{K}_{ijhk}]\cdot[\boldsymbol{\delta}_{hk}]$$

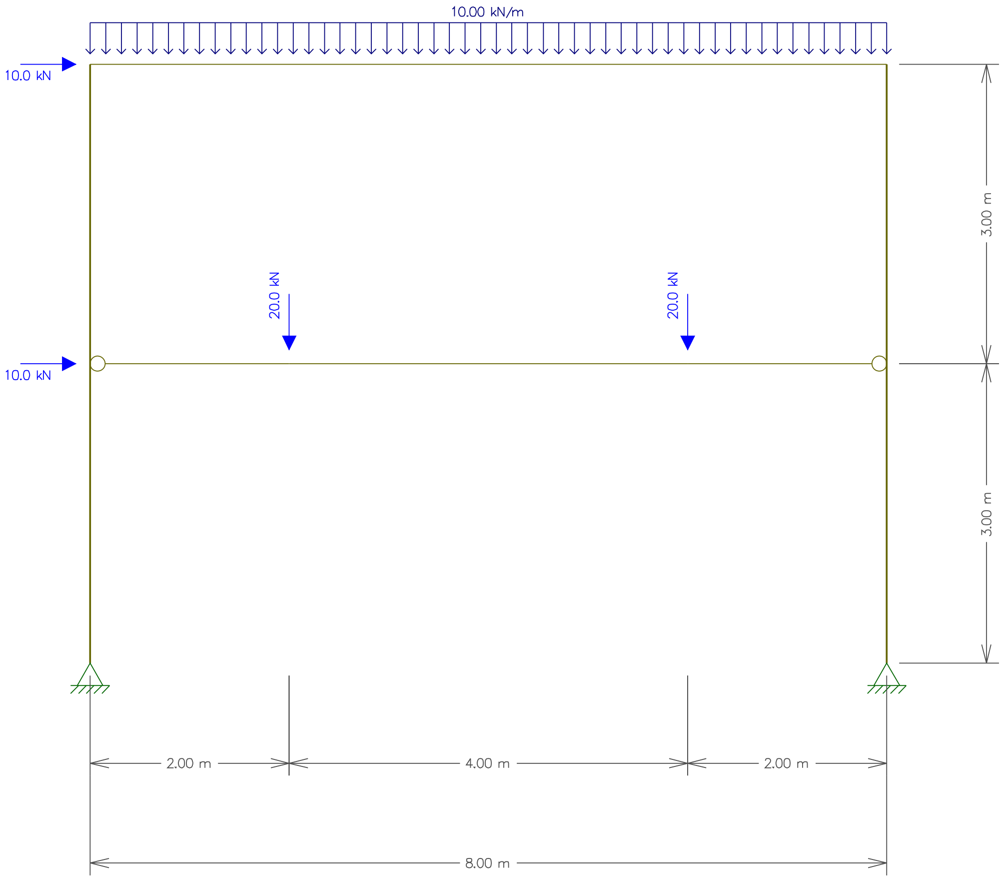
$$
\begin{bmatrix} 0\\ -F\\ \dfrac{Q\cdot L^2}{12}\\ -F\\ 0\\ 0\\ -\dfrac{Q\cdot L^2}{12}\\ 0 \end{bmatrix}
=
\begin{bmatrix}
7K_1 & 3K_2 & 2K_1 & -6K_2 & 0 & 0 & 0 & 0\\
-3K_2 & -[15K_4+K_5] & -6K_2 & 12K_4 & 0 & K_5 & 0 & 0\\
2K_1 & 6K_2 & 4[K_1+K_3] & -6K_2 & 0 & 0 & -2K_3 & 0\\
6K_2 & 12K_4 & 6K_2 & -[12K_4+K_5] & 0 & 0 & 0 & K_5\\
0 & 0 & 0 & 0 & 7K_1 & 3K_2 & -2K_1 & -6K_2\\
0 & K_5 & 0 & 0 & -3K_2 & -[15K_4+K_5] & 6K_2 & 12K_4\\
0 & 0 & 2K_3 & 0 & 2K_1 & 6K_2 & -4[K_1+K_3] & -6K_2\\
0 & 0 & 0 & K_5 & 6K_2 & 12K_4 & -6K_2 & -[12K_4+K_5]
\end{bmatrix}
\cdot
\begin{bmatrix} \varphi_1\\ \Delta_1\\ \varphi_2\\ \Delta_2\\ \varphi_3\\ \Delta_3\\ \varphi_4\\ \Delta_4 \end{bmatrix}
$$

La soluzione risulta:

$[\boldsymbol{\delta}_{hk}]=[\mathbf{K}_{ijhk}]^{-1}\cdot[\mathbf{F}_{ij}]$ valida se e solo se $\det[\mathbf{K}_{ijhk}]\neq 0$ ovvero se la matrice di rigidezza è invertibile.

Per la soluzione si considerino i seguenti valori geometrici e meccanici:

Modulo di elasticità del calcestruzzo:	$E = 3 \cdot 10^7$	[kN/m²]
Sezione trasversale dei pilastri:	$A_c = 0.4^2$	[m²]
Sezione trasversale delle travi:	$A_t = 0.3 \cdot 0.6$	[m²]



$$K_1 = \frac{EI_c}{h} = \frac{3 \cdot 10^7 \cdot \left[\frac{0.4^4}{12}\right]}{3} = 21333$$ [kNm]

$$K_2 = \frac{EI_c}{h^2} = \frac{3 \cdot 10^7 \cdot \left[\frac{0.4^4}{12}\right]}{9} = 7111$$ [kN]

$$K_3 = \frac{EI_t}{L} = \frac{3 \cdot 10^7 \cdot \left[\frac{0.3 \cdot 0.6^3}{12}\right]}{8} = 20250$$ [kNm]

$$K_4 = \frac{EI_c}{h^3} = \frac{3 \cdot 10^7 \cdot \left[\frac{0.4^4}{12}\right]}{27} = 2370$$ [kN/m]

$$K_5 = \frac{EA_t}{L} = \frac{3 \cdot 10^7 \cdot [0.3 \cdot 0.6]}{8} = 675000$$ [kN/m]

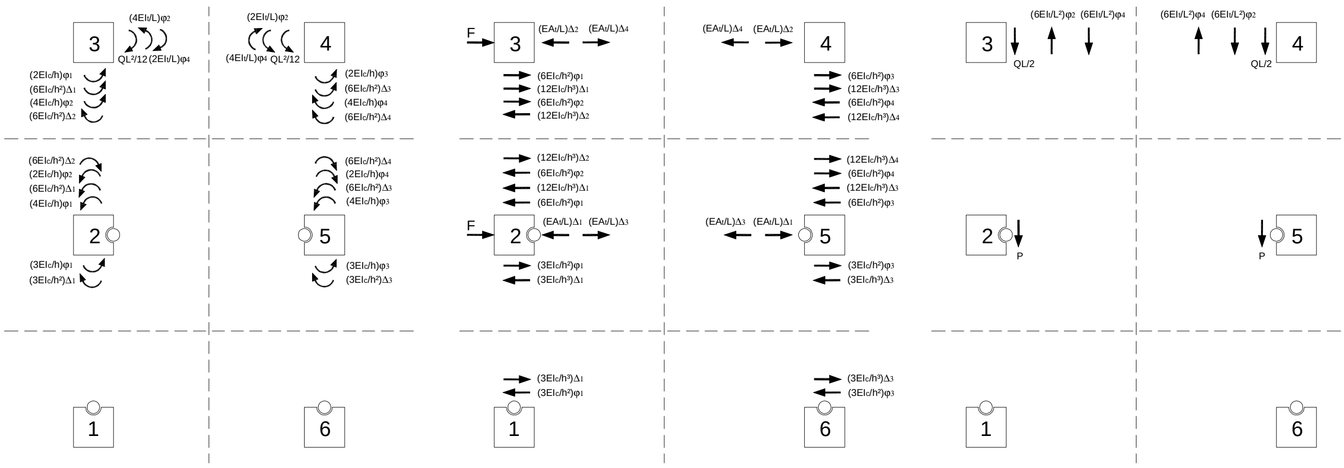
$$
\begin{bmatrix} 0 \\ -F \\ \dfrac{Q \cdot L^2}{12} \\ -F \\ 0 \\ 0 \\ -\dfrac{Q \cdot L^2}{12} \\ 0 \end{bmatrix}
=
\begin{bmatrix}
7K_1 & 3K_2 & 2K_1 & -6K_2 & 0 & 0 & 0 & 0 \\
-3K_2 & -[15K_4+K_5] & -6K_2 & 12K_4 & 0 & K_5 & 0 & 0 \\
2K_1 & 6K_2 & 4[K_1+K_3] & -6K_2 & 0 & 0 & -2K_3 & 0 \\
6K_2 & 12K_4 & 6K_2 & -[12K_4+K_5] & 0 & 0 & 0 & K_5 \\
0 & 0 & 0 & 0 & 7K_1 & 3K_2 & -2K_1 & -6K_2 \\
0 & K_5 & 0 & 0 & -3K_2 & -[15K_4+K_5] & 6K_2 & 12K_4 \\
0 & 0 & 2K_3 & 0 & 2K_1 & 6K_2 & -4[K_1+K_3] & -6K_2 \\
0 & 0 & 0 & K_5 & 6K_2 & 12K_4 & -6K_2 & -[12K_4+K_5]
\end{bmatrix}
\cdot
\begin{bmatrix} \varphi_1 \\ \Delta_1 \\ \varphi_2 \\ \Delta_2 \\ \varphi_3 \\ \Delta_3 \\ \varphi_4 \\ \Delta_4 \end{bmatrix}
$$

$$
\begin{bmatrix} 0 \\ -10 \\ 53.34 \\ -10 \\ 0 \\ 0 \\ -53.34 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
149331 & 21333 & 42666 & -42666 & 0 & 0 & 0 & 0 \\
-21333 & -710550 & -42666 & 28440 & 0 & 675000 & 0 & 0 \\
42666 & 42666 & 166332 & -42666 & 0 & 0 & -40500 & 0 \\
42666 & 28440 & 42666 & -703440 & 0 & 0 & 0 & 675000 \\
0 & 0 & 0 & 0 & 149331 & 21333 & -42666 & -42666 \\
0 & 675000 & 0 & 0 & -21333 & -710550 & 42666 & 28440 \\
0 & 0 & 40500 & 0 & 42666 & 42666 & -166332 & -42666 \\
0 & 0 & 0 & 675000 & 42666 & 28440 & -42666 & -703440
\end{bmatrix}
\cdot
\begin{bmatrix} \varphi_1 \\ \Delta_1 \\ \varphi_2 \\ \Delta_2 \\ \varphi_3 \\ \Delta_3 \\ \varphi_4 \\ \Delta_4 \end{bmatrix}
$$

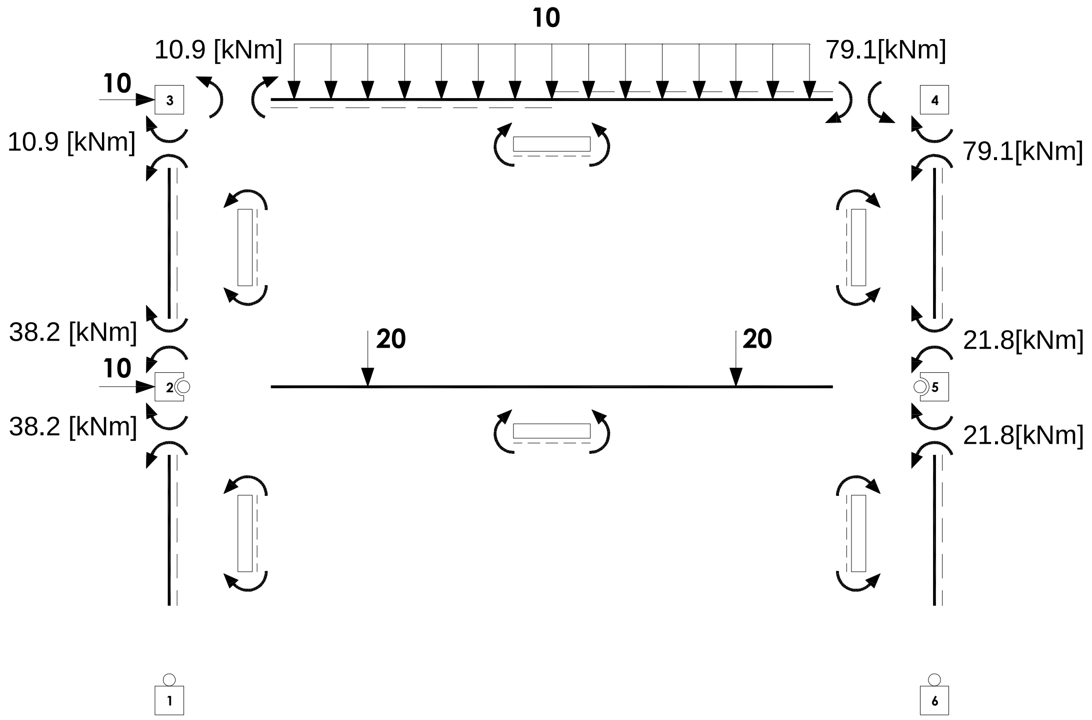
La soluzione del sistema si può ottenere con il Metodo di Cramer:

$$
\begin{bmatrix} \varphi_1 \\ \Delta_1 \\ \varphi_2 \\ \Delta_2 \\ \varphi_3 \\ \Delta_3 \\ \varphi_4 \\ \Delta_4 \end{bmatrix}
=
\begin{bmatrix} 2.000 \cdot 10^{-3} \\ 7.792 \cdot 10^{-3} \\ 8.469 \cdot 10^{-4} \\ 1.174 \cdot 10^{-2} \\ 2.262 \cdot 10^{-3} \\ 7.809 \cdot 10^{-3} \\ 1.051 \cdot 10^{-4} \\ 1.172 \cdot 10^{-4} \end{bmatrix}
\begin{matrix} [rad] \\ [m] \\ [rad] \\ [m] \\ [rad] \\ [m] \\ [rad] \\ [m] \end{matrix}
$$

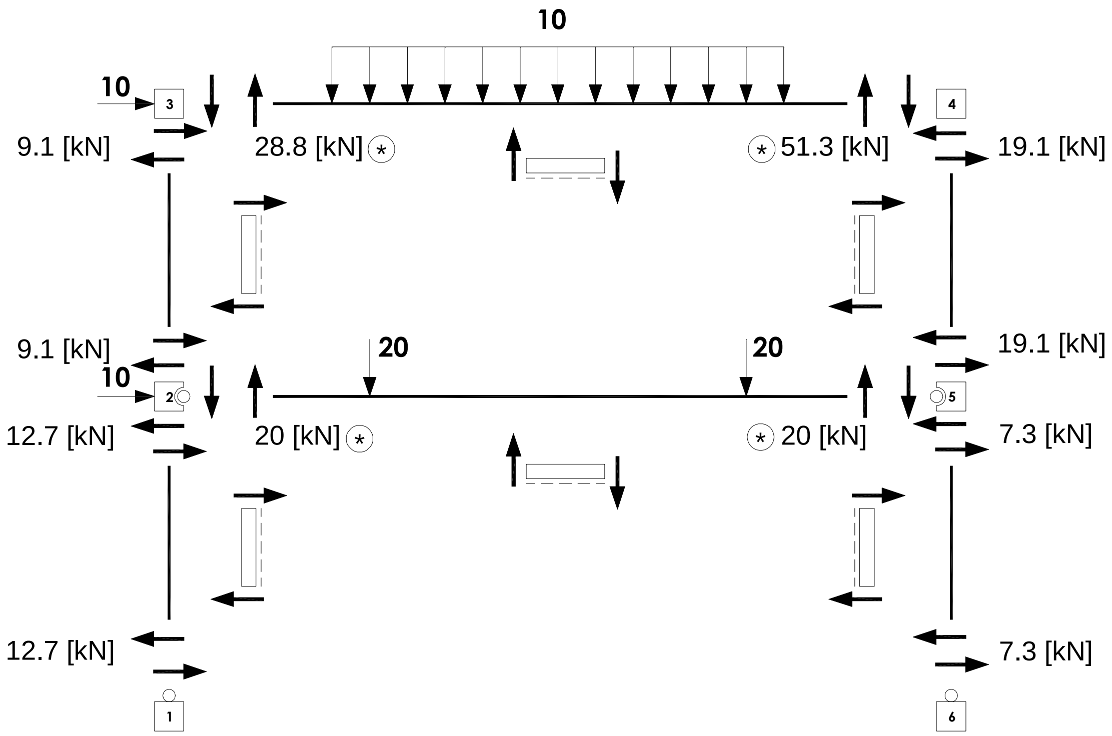
Sostituendo il valore degli spostamenti nell'equilibrio dei nodi, si ottiene il sistema equilibrato con le azioni esterne:



Momenti



Tagli



Normali

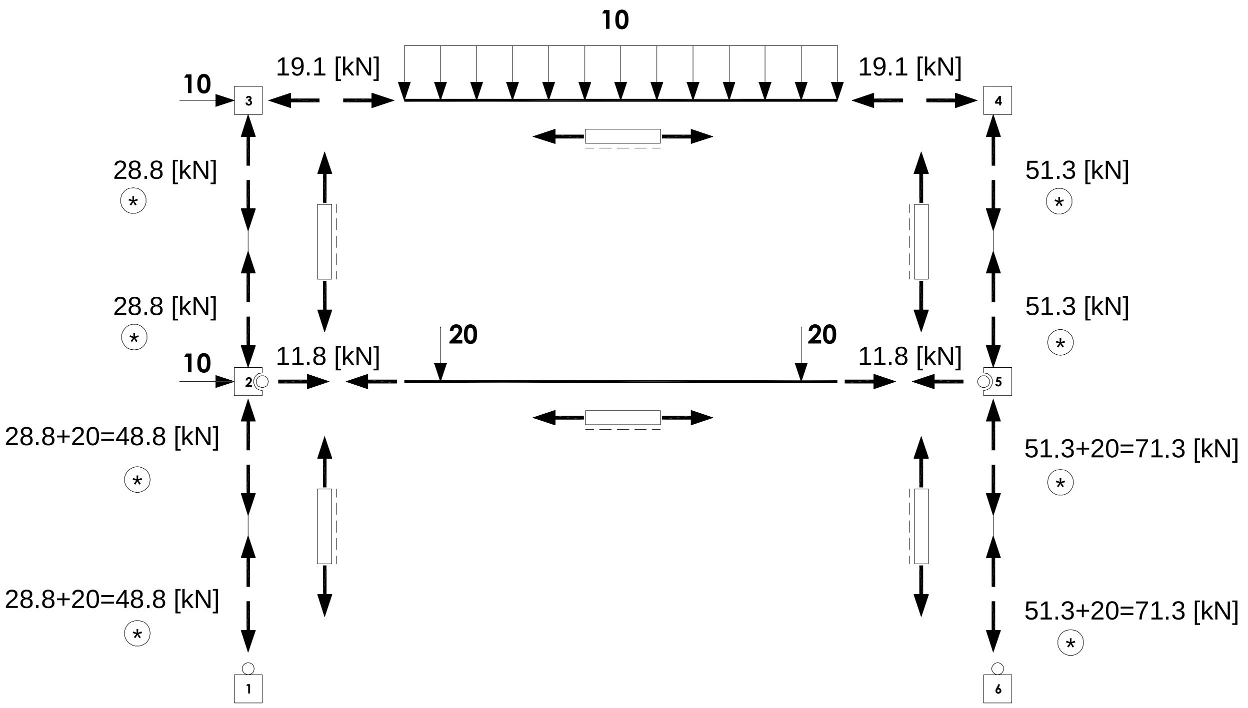


Diagramma dei Momenti Flettenti

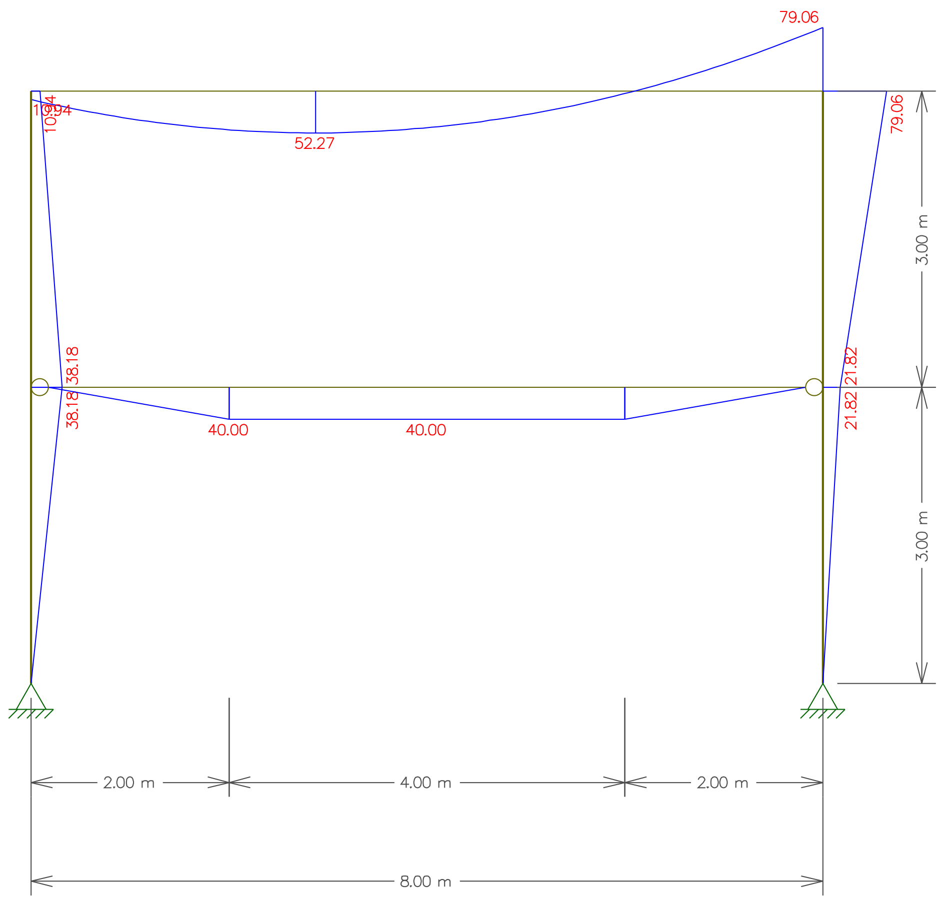


Diagramma del Taglio

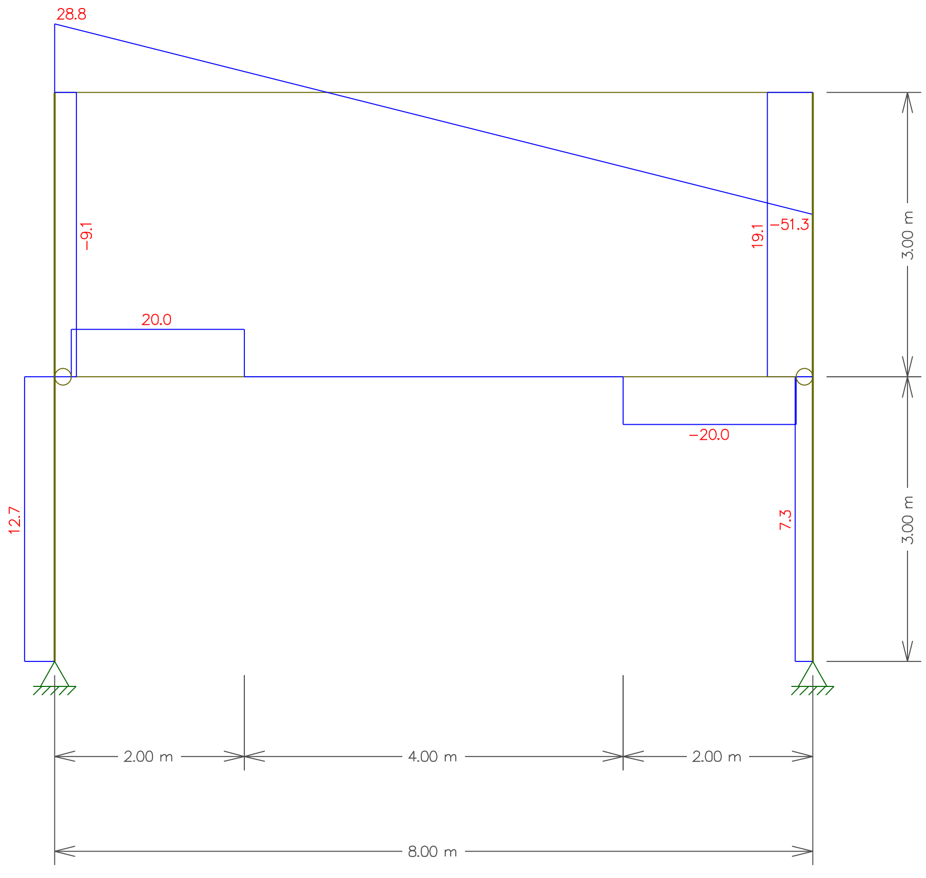
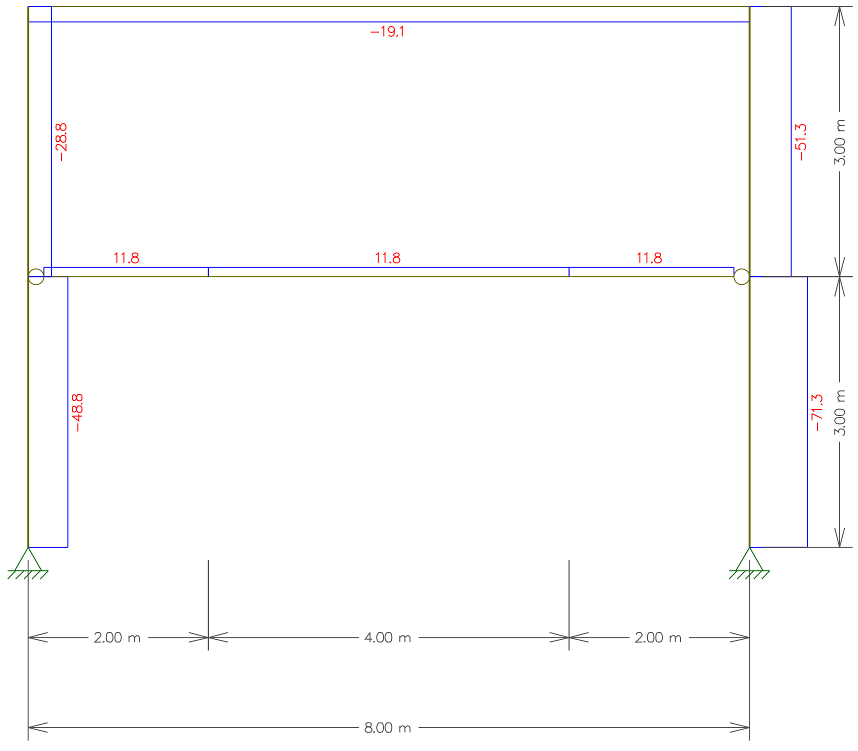


Diagramma della Normale



Deformata Qualitativa